

2.5 Apply the Remainder and Factor Theorems

Monday, December 4, 2017 7:12 AM

Polynomial Long Division

Divide $f(x) = 3x^4 - 5x^3 + 4x - 6$ by $x^2 - 3x + 5$

$$\begin{array}{r}
 \overline{) 3x^4 - 5x^3 + 0x^2 + 4x - 6} \\
 \underline{- (3x^4 - 9x^3 + 15x^2)} \\
 4x^3 - 15x^2 + 4x \\
 \underline{- (4x^3 - 12x^2 - 20x)} \\
 -3x^2 - 16x - 6 \\
 \underline{- (-3x^2 + 9x - 15)} \\
 -25x + 9
 \end{array}$$

$$3x^2 + 4x - 3 + \frac{-25x + 9}{x^2 - 3x + 5}$$

Synthetic Division

Synthetic division can be used to divide any polynomial by a divisor of the form $x - k$

Divide $f(x) = x^3 + 5x^2 - 7x + 2$ by $x - 2$

$$\begin{array}{r|rrrr}
 2 & 1 & 5 & -7 & 2 \\
 & & 2 & 14 & 14 \\
 \hline
 & 1 & 7 & 7 & 16
 \end{array}$$

$$\begin{array}{cccc}
 1 & 7 & 7 & 16 \\
 \swarrow & \uparrow & \nearrow & \uparrow \\
 \text{Coefficients} & & & \text{Remainder}
 \end{array}$$

$$x^2 + 7x + 7 + \frac{16}{x - 2}$$

Factor a Polynomial

Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $(x + 2)$ is a factor

$$\begin{array}{r|rrrr}
 -2 & 3 & -4 & -28 & -16 \\
 & & -6 & 20 & 16 \\
 \hline
 & 3 & -10 & -8 & 0
 \end{array}
 \leftarrow \text{Remainder Must be Zero}$$

$(x + 2)(3x^2 - 10x - 8)$	AC Method
$(3x^2 - 12x)(2x - 8)$	GCF each group
$3x(x - 4) + 2(x - 4)$	GCF again
$(x + 2)(x - 4)(3x + 2)$	